Period 3, Oct 18, 2024

1) $y = (7x^3 - 5x^2 + 6x)^8 \Rightarrow y = u^8$

$u = 7x^3 - 5x^2 + 6x$ $\qquad \frac{dy}{du} = 8u^7$

$\frac{du}{dx} = 21x^2 - 10x + 6$

$\frac{du}{dx} \cdot \frac{dy}{du} = (21x^2 - 10x + 6) \cdot 8 \cdot (7x^3 - 5x^2 + 6x)^7$

correct $= 8(21x^2 - 10x + 6)(7x^3 - 5x^2 + 6x)^7$

$= (168x^2 - 80x + 48)(7x^3 - 5x^2 + 6x)^7$

$\cancel{\text{wrong} = (21x^2 - 10x + 6)(56x^3 - 40x^2 + 48x)^7}$

$5(x+y)^2 \neq (5x+5y)^2$

$5(x+y)(x+y) \neq (5x+5y)(5x+5y)$

$5(x^2 + 2xy + y^2) \neq 25x^2 + 25xy + 25xy + 25y^2$

$5x^2 + 10xy + 5y^2 \neq 25x^2 + 50xy + 25y^2$

$y = (5x^4 - 5x^3 + 6x)^3 \Rightarrow y = u^3$

$u = 5x^4 - 5x^3 + 6x$ $\qquad \frac{dy}{du} = 3u^2$

$\frac{du}{dx} = 20x^3 - 15x^2 + 6$

$\frac{du}{dx} \cdot \frac{dy}{du} = \frac{dy}{dx}$

$(20x^3 - 15x^2 + 6)(3u^2) = 3(20x^3 - 15x^2 + 6)(5x^4 - 5x^3 + 6x)^2$ ← correct

$\cancel{\text{wrong} = (20x^3 - 15x^2 + 6)(15x^4 - 15x^3 + 18x)^2}$

$$y = (2x^3 - 5x^2 + 6x)^5 \Rightarrow y = u^5$$

$u = 2x^3 - 5x^2 + 6x$ $\qquad \frac{dy}{du} = 5u^4$

$$\frac{du}{dx} = 6x^2 - 10x + 6$$

$$\frac{\cancel{du}}{dx} \cdot \frac{dy}{\cancel{du}} = \frac{dy}{dx}$$

$$(6x^2 - 10x + 6)(5u^4) = 5(6x^2 - 10x + 6)(2x^3 - 5x^2 + 6x)^4$$
$$= (30x^2 - 50x + 30)(2x^3 - 5x^2 + 6x)^4$$

$$y = \sin^4(3x^2 + 6x + 4) \Rightarrow y = \sin^4 u \Rightarrow y = L^4$$

$u = 3x^2 + 6x + 4$

$\frac{du}{dx} = 6x + 6$ $\qquad L = \sin u$, $\frac{dL}{du} = \cos u$ $\qquad \frac{dy}{dL} = 4L^3$

$$\frac{\cancel{du}}{dx} \cdot \frac{\cancel{dL}}{\cancel{du}} \cdot \frac{dy}{\cancel{dL}} = (6x+6)(\cos u)4L^3 = 4(6x+6)\cos(3x^2+6x+4)\sin^3(3x^2+6x+4)$$

correct $= 4(6x+6)[\cos(3x^2+6x+4)][\sin(3x^2+6x+4)]^3$

$\neq$

wrong $= 4(6x+6)[\cos(3x^2+6x+4)][\sin(3x^2+6x+4)^3]$

$y = \sin^5(x^5+6x^3+4) \Rightarrow y = \sin^5 u \Rightarrow y = L^5$

$u = x^5+6x^3+4$ $\qquad L = \sin u$ $\qquad \frac{dy}{dL} = 5L^4$

$\frac{du}{dx} = 5x^4+18x^2$ $\qquad \frac{dL}{du} = \cos u$

$$\frac{\cancel{du}}{dx}\cdot\frac{\cancel{dL}}{\cancel{du}}\cdot\frac{dy}{\cancel{dL}}$$

$$(5x^4+18x^2)(\cos u)(5L^4) = \frac{dy}{dx}$$

$$\frac{dy}{dx} = 5(5x^4+18x^2)[\cos(x^5+6x^3+4)][\sin^4(x^5+6x^3+4)]$$

$$= 5(5x^4+18x^2)[\cos(x^5+6x^3+4)][\sin(x^5+6x^3+4)]^4$$

$$= (25x^4+90x^2)[\cos(x^5+6x^3+4)][\sin^4(x^5+6x^3+4)]^4$$

$$\neq$$

wrong $= (25x^4+90x^2)[\cos(x^5+6x^3+4)][\sin(x^5+6x^3+4)^4]$

$y = \sin^7(x^3+6x^2+4) \Rightarrow y = \sin^7 u \Rightarrow y = L^7$

$u = x^3+6x^2+4$ $\qquad L = \sin u$ $\qquad \frac{dy}{dL} = 7L^6$

$\frac{du}{dx} = 3x^2+12x$ $\qquad \frac{dL}{du} = \cos u$

$$\frac{\cancel{du}}{dx}\cdot\frac{\cancel{dL}}{\cancel{du}}\cdot\frac{dy}{\cancel{dL}} = \frac{dy}{dx}$$

$$(3x^2+12x)(\cos u)(7L^6) =$$

$$\frac{dy}{dx} = 7(3x^2+12x)[\cos(x^3+6x^2+4)][\sin^6(x^3+6x^2+4)$$

105, ~~71~~, 107, ~~10~~, ~~49~~, ~~77~~, 81, 83, ~~37~~, ~~25~~

10.

$$h(x) = \frac{3x}{8} - 2x^2 = \frac{3}{8}x - 2x^2$$

$$\left(-2, \frac{-35}{4}\right)$$

$$h'(x) = \frac{3}{8} - 4x$$

$$h'(-2) = \frac{3}{8} - 4(-2) = \frac{3}{8} + 8 = \frac{3}{8} + \frac{64}{8} = \frac{67}{8} = \text{Slope of Tangent Line}$$

$\left(-2, \frac{-35}{4}\right)$ point

25.

$$g(T) = \frac{2}{3T^2} = \frac{2}{3}T^{-2}$$

$$g'(T) = \frac{\overset{0}{\cancel{0(3T^2)}} - 2 \cdot 6T}{(3T^2)^2} = \frac{-12T}{9T^4} = \frac{-\cancel{3} \cdot 4 \cdot \cancel{T}}{\cancel{3} \cdot 3 \cdot \cancel{T} \cdot T^3} = \frac{-4}{3T^3}$$

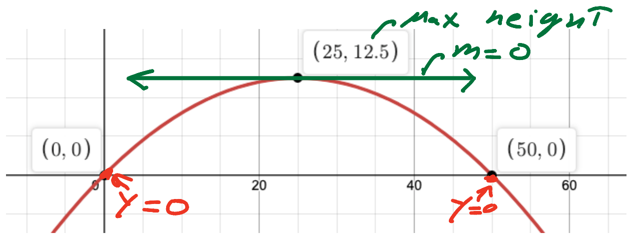
$$g(T) = \frac{2}{3}T^{-2}$$

$$g'(T) = \frac{2}{3} \cdot -2\, T^{-2-1} = \frac{-4T^{-3}}{3} = \frac{-4}{3T^3}$$

Same

$y = x - 0.02x^2$

Max height
Slope $= 0$



$\frac{dy}{dx} = 1 - 0.02 \cdot 2x^{2-1} = 1 - 0.04x^1$

Max height

$0 = 1 - \cancel{0.04x}$

$0.04x \quad +\cancel{0.04x}$

$\frac{0.04x}{0.04} = \frac{1}{0.04}$

$x = 25$ where max height happens.

Total distance
Lands when $y=0$

$0 = x - 0.02x^2$

$0 = x(1 - 0.02x)$

$x = 0$ or $0 = 1 - 0.02x$

$+0.02x \quad +\cancel{0.02x}$

$\frac{0.02x}{0.02} = \frac{1}{0.02} \rightarrow x = 50$

To Find Max height

$y = 25 - 0.02(25)^2$

$y = 25 - 0.02(625)$

$y = 25 - 12.5 = 12.5$

IRC

$\frac{dy}{dx} = 1 - 0.04x$

x	$\frac{dx}{dx}$
0	$1 = 1 - 0.04(0) = 1 - 0$ going up
10	$.6 = 1 - 0.04(10) = 1 - .04$ going up
25	$0 = 1 - 0.04(25) = 1 - 1$ max height
30	$-.2 = 1 - 0.04(30) = 1 - 1.2$ going down
50	$-1 = 1 - 0.04(50) = 1 - 2$ going down/Landing

$y = \frac{x^4}{\cos x}$

$$\frac{dy}{dx} = \frac{4x^3(\cos x) - x^4(-\sin x)}{(\cos x)^2} = \frac{x^3[4\cos x + x\sin x]}{(\cos x)^2} = \frac{x^3[4\cos x + x\sin x]}{\cos^2 x}$$

71, 77

$F(x) = \underbrace{(x^2-1)^{5/2}}\,\underbrace{(x^3+5)}$

$F'(x) = \left[\frac{d}{dx}\left[(x^2-1)^{5/2}\right]\right](x^3+5) + (x^2-1)^{5/2}(3x^2)$

$y = (x^2-1)^{5/2} \Rightarrow y = u^{5/2}$

$u = x^2-1$

$\frac{du}{dx} = 2x$

$\frac{dy}{du} = \frac{5}{2}u^{\frac{3}{2}}$

$\frac{\cancel{du}}{dx}\cdot\frac{dy}{\cancel{du}} = \frac{dy}{dx}$

$\cancel{2}x\cdot\frac{5}{\cancel{2}}u^{3/2} = 5x(x^2-1)^{3/2}$

$F'(x) = 5x(x^2-1)^{3/2}(x^3+5) + (x^2-1)^{5/2}(3x^2)$

$y = \frac{2}{3}\sin^{3/2}x - \frac{2}{7}\sin^{\frac{7}{2}}x$

$\frac{dy}{dx} = \cancel{\frac{2}{3}}\cdot\cancel{\frac{3}{2}}\cos x\sin^{\frac{1}{2}}x - \cancel{\frac{2}{7}}\cancel{\frac{7}{2}}\cos x\sin^{\frac{5}{2}}x$

$\boxed{\frac{dy}{dx} = \cos x\left(\sin^{\frac{1}{2}}x - \sin^{5/2}x\right)}$

$y = \sin^{7/2}x \Rightarrow y = u^{\frac{7}{2}}$

$u = \sin x$

$\frac{du}{dx} = \cos x$

$\frac{dy}{du} = \frac{7}{2}u^{\frac{7}{2}-1} = \frac{7}{2}u^{5/2}$

$\frac{du}{dx} = \cos x\cdot\frac{7}{2}(\sin x)^{5/2}$

$y = \sin^{3/2}x$

$\frac{dy}{dx} = \frac{3}{2}\cos x\cdot\sin^{\frac{1}{2}}x$

81, 83, 105, 107

81. $y = \sqrt{1-x^3}$ $\quad (-2, 3)$

$u = 1 - x^3$ $\quad y = \sqrt{u} = u^{\frac{1}{2}}$

$\frac{du}{dx} = -3x^2$ $\quad \frac{dy}{du} = \frac{1}{2}u^{\frac{1}{2}-1} = \frac{1}{2}u^{-\frac{1}{2}} = \frac{1}{2\sqrt{u}}$

$\frac{\cancel{du}}{dx} \cdot \frac{dy}{\cancel{du}} = \frac{dy}{dx}$

$\frac{dy}{dx} = -3x^2\left(\frac{1}{2\sqrt{1-x^3}}\right) = \frac{-3x^2}{2\sqrt{1-x^3}}$ $= \frac{-3(-2)^2}{2\sqrt{1-(-2)^3}} = \frac{-3 \cdot 4}{2\sqrt{1-(-8)}}$

$= \frac{-12}{2\sqrt{1+8}} = \frac{-12}{2\sqrt{9}} = \frac{-12}{2 \cdot 3}$

83. $y = \frac{1}{2}\csc 2x$ $\quad \left(\frac{\pi}{4}, \frac{1}{2}\right)$

$u = 2x$ $\quad y = \frac{1}{2}\csc u$

$\frac{du}{dx} = 2$ $\quad \frac{dy}{du} = \frac{1}{2}(-\csc u \cot u)$

$\frac{\cancel{du}}{dx} \cdot \frac{dy}{\cancel{du}} = \frac{dy}{dx}$

$\cancel{2 \cdot \frac{1}{2}}(-\csc 2x \cot 2x)$

$-\csc\frac{\pi}{2}\cot\frac{\pi}{2} = -\frac{1}{\sin\frac{\pi}{2}} \cdot \frac{\cos\frac{\pi}{2}}{\sin\frac{\pi}{2}}$

$= \frac{-1}{1} \cdot \frac{0}{1} = \frac{0}{1} = 0$

$2 \cdot \frac{\pi}{4} = \frac{2\pi}{4} = \frac{\pi}{2}$

$\sin\frac{\pi}{2} = 1$

$\cos\frac{\pi}{2} = 0$

$\csc 2x = \frac{1}{\sin 2x}$

$\cot 2x = \frac{\cos 2x}{\sin 2x}$

105.

$$\sqrt{xy} = x - 4y$$

$$\sqrt{x}\cdot\sqrt{y} = x - 4y$$

$$x^{\frac{1}{2}}\cdot y^{\frac{1}{2}} = x - 4y$$

$$\frac{1}{2\sqrt{x}}\cdot y^{\frac{1}{2}} + x^{\frac{1}{2}}\left(\frac{1}{2\sqrt{y}}\cdot\frac{dy}{dx}\right) = 1 - 4\frac{dy}{dx}$$

$$\cancel{\frac{\sqrt{y}}{2\sqrt{x}}} + \frac{\sqrt{x}}{2\sqrt{y}}\cdot\frac{dy}{dx} = 1 - 4\cancel{\frac{dy}{dx}}$$

$$-\frac{\sqrt{y}}{2\sqrt{x}} \qquad +4\frac{dy}{dx}$$

$$+4\frac{dy}{dx} \quad -\frac{\sqrt{y}}{2\sqrt{x}}$$

$$4\frac{dy}{dx} + \frac{\sqrt{x}}{2\sqrt{y}}\frac{dy}{dx} = 1 - \frac{\sqrt{y}}{2\sqrt{x}}$$

$$\frac{\frac{dy}{dx}\left(\cancel{4 + \frac{\sqrt{x}}{2\sqrt{y}}}\right)}{\cancel{4 + \frac{\sqrt{x}}{2\sqrt{y}}}} = \frac{1 - \frac{\sqrt{y}}{2\sqrt{x}}}{4 + \frac{\sqrt{x}}{2\sqrt{y}}}$$

$$\frac{dy}{dx} = \frac{\frac{2\sqrt{x}}{2\sqrt{x}} - \frac{\sqrt{y}}{2\sqrt{x}}}{\frac{8\sqrt{y}}{2\sqrt{y}} + \frac{\sqrt{x}}{2\sqrt{y}}} = \frac{\frac{2\sqrt{x} - \sqrt{y}}{2\sqrt{x}}}{\frac{8\sqrt{y} + \sqrt{x}}{2\sqrt{y}}}$$

$$\frac{2\sqrt{x} - \sqrt{y}}{\cancel{2}\sqrt{x}}\cdot\frac{\cancel{2}\sqrt{y}}{8\sqrt{y} + \sqrt{x}}$$

$$\frac{\sqrt{y}\,(2\sqrt{x} - \sqrt{y})}{\sqrt{x}\,(8\sqrt{y} + \sqrt{x})}$$

$$x\sin y = y\cos x$$

$$1\cdot\sin y + x\cdot\cos y\,\frac{dy}{dx} = \frac{dy}{dx}\cdot\cos x + y\cdot(-\sin x)$$

$$\sin y + \cancel{x\cos y\,\frac{dy}{dx}} = \cos x\,\frac{dy}{dx} \cancel{- y\sin x}$$

$$+\,y\sin x \qquad\qquad + \cancel{y\sin x}$$

$$-\cancel{x\cos y\,\frac{dy}{dx}} \qquad\qquad - x\cos y\,\frac{dy}{dx}$$

$$\frac{\sin y + y\sin x}{\cos x - x\cos y} = \frac{dy}{dx}\,\frac{\cancel{(\cos x - x\cos y)}}{\cancel{\cos x - x\cos y}}$$

$$\frac{\sin y + y\sin x}{\cos x - x\cos y} = \frac{dy}{dx}$$